

A connection between gravity and the Higgs field

M. Consoli

Istituto Nazionale di Fisica Nucleare, Sezione di Catania
Corso Italia 57, 95129 Catania, Italy

Abstract

Several arguments suggest that an effective curved space-time structure (of the type as in General Relativity) can actually find its dynamical origin in an underlying condensed medium of spinless quanta. For this reason, we exploit the recent idea of density fluctuations in a ‘Higgs condensate’ with the conclusion that such long-wavelength effects might represent the natural dynamical agent of gravity.

1 Introduction

Following the original induced-gravity models [1], one may attempt to describe gravity as an effective force induced by the vacuum structure, in analogy with the attractive interaction among electrons that can only exist in the presence of an ion lattice. If such an underlying ‘dynamical’ mechanism is found, one will also gain a better understanding of those peculiar ‘geometrical’ properties that are the object of classical General Relativity. In fact, by demanding the space-time structure to an undefined energy-momentum tensor, no clear distinction between the two aspects is possible and this may be the reason for long-standing problems (e.g. the space-time curvature associated with the vacuum energy).

A nice example to understand what aspects may be kinematical and what other aspects may be dynamical has been given by Visser [2]. Namely, a curved space-time pseudo-Riemannian geometry arises when studying the density fluctuations in a moving irrotational fluid, i.e. where the velocity field is the gradient of a scalar potential $\sigma(x)$. In this system, the underlying space-time is exactly flat but the propagation of long-wavelength fluctuations is governed by a curved ‘acoustic’ metric $g_{\mu\nu}(x)$ determined at each space-time point x by the physical parameters of the fluid (density, velocity and pressure). These can all be expressed in terms of $\sigma(x)$ through the hydrodynamical equations. In this example, there is a system, the fluid, whose constituents are governed by some underlying molecular interactions that, on macroscopic scales, can be summarized in the value of a scalar field $\sigma(x)$. At some intermediate level, this contains the dynamical informations. On the other hand, the effective metric tensor $g_{\mu\nu}$ is a purely kinematical quantity that depends on $\sigma(x)$ in a general parametric form

$$g_{\mu\nu}(x) = g_{\mu\nu}[\sigma(x)] \quad (1)$$

Notice that there is no non-trivial curvature in the equilibrium state $\sigma(x) = \sigma_o = \text{const.}$ where any fluid is *self-sustaining* [3].

Looking for the physical origin of gravity, one should also look at ref. [4]. It contains an exhaustive set of many gravity-analogs, namely of systems that can simulate and/or reproduce the experimental properties of gravitation (moving fluids, condensed matter systems with a refractive index, Bose-Einstein condensates,...). Again the hydrodynamical analogy, when exploited in a lagrangian field-theoretical context, leads to the emergence of a curved, effective geometry as a result of linearizing a scalar field theory (in flat space-time) around some non-trivial background configuration. For this reason, one may agree with the authors of ref.[4] and conclude that General Relativity provides some kind of universal low-energy picture, just

as hydrodynamics. This, concentrating on the properties of matter at scales that are much larger than the mean free path for the elementary constituents, is insensitive to the details of short-distance molecular dynamics. This point of view should not sound too surprising to the extent that is consistent with the historical origin of General Relativity before the birth of quantum mechanics.

Notice that we are not saying that one can *prove* a formal equivalence with Einstein's General Relativity. In fact, a medium provides a definite physical framework whose short-distance details might have no counterpart in General Relativity. However, by restricting to long-wavelength phenomena, and to some degree of accuracy, the two descriptions may be indistinguishable. At the same time, there may be different dynamical scenarios at very small scales that, however, become equivalent to General Relativity on a larger scale. Their number can only be restricted by exploiting this type of correspondence, within the constraints of the experimental results. For instance, $\sigma(x)$, rather than arising from a fundamental scalar theory, might turn out to be a collective excitation of a superfluid fermionic vacuum [3].

On the other hand, the idea of an 'Aethereal Medium', whose density variations could account for the gravitational force, is deeply rooted in the origin of our scientific culture and can be found, for instance, in Newton himself. In fact, although refraining from a definite hypothesis on the physical origin of gravity in the *Principia* (".. I frame no hypotheses..and to us it is enough that gravity does really exist, and act according to the laws which we have explained .."), Newton explicitly inserted some additional *Queries on the Aether* in the last version of *Opticks*. In particular, in Query 21, he was considering that "...if the elastick force of this Medium be exceeding great, it may suffice to impel Bodies from the denser parts of the Medium towards the rarer, with all that power which we call Gravity". To this end, the hypothetical aetherial particles should feel a repulsive force ("...Particles which endeavour to recede from one another..") and be "...exceedingly smaller than those of Air, or even than those of Light..." thus making "...that Medium exceedingly more rare and elastick than Air..".

It should not be too difficult, at least for a present-day particle physicist, to realize that the required properties of this hypothetical medium might have a well defined counterpart in the quantum vacuum of a spontaneously broken $\lambda\Phi^4$ theory: the 'Higgs condensate'. In fact, the name itself means that a non-vanishing expectation value $\langle\Phi\rangle \neq 0$ should correspond to some kind of medium, an aether, made up by the physical condensation process of elementary spinless quanta, the 'phions' [5], whose 'empty' vacuum state is not the true ground state of the theory.

The symmetric phase, where $\langle\Phi\rangle = 0$, will eventually be re-established at a phase tran-

sition temperature $T = T_c$. This, in the Standard Model, is so high that we can safely approximate the scalar condensate as a zero-temperature system. This observation provides the argument to represent the pion condensate as a medium where bodies flow without any apparent friction, as superfluid ^4He . In fact, a zero-temperature Bose system is a ‘quantum liquid’, i.e. a system whose macroscopic properties depend on the quantum nature of its constituents. This requires a form of quantum hydrodynamics, of the type originally considered by Landau [6], where the local density of the fluid $n(\mathbf{r})$ and the current density vector $\mathbf{J}(\mathbf{r})$ have canonical commutation relations [8], as in quantum mechanics for the position and momentum operators.

The analogy with superfluid helium is also supported by the other observation [5] that, as for the interatomic ^4He - ^4He potential, the low-energy limit of cutoff $\lambda\Phi^4$ is a theory of spinless quanta with a short-range repulsive core and a long-range attractive tail. The latter originates from ultraviolet-finite parts of higher loop graphs [5] that give rise to a $-\frac{\lambda^2 e^{-2m_\Phi r}}{r^3}$ attraction where m_Φ is the pion mass. Differently from the usual ultraviolet divergences, this ultraviolet-finite part cannot be reabsorbed into a standard re-definition of the tree-level, repulsive $+\lambda\delta^{(3)}(\mathbf{r})$ potential and is essential for a physical description of the condensation process when approaching the phase transition limit $m_\Phi \rightarrow 0$ where the symmetric vacuum at $\langle\Phi\rangle = 0$ becomes unstable.

Although representing some kind of aether, the scalar condensate is, however, different from the aether of classical physics and there are phenomena that cannot be understood at the classical level. For instance, since the superfluid flow is a potential flow, there is no drag force on a body moving in the fluid and no D’Alembert paradox [9] as if the fluid were locally dragged by all moving bodies. In this sense, an origin of gravity from the phenomenon of vacuum condensation would require only a modest switch from Einstein’s Special Relativity to Lorentzian Relativity, with the local gravity field providing the operative definition of Lorentz’s aether [10].

This point of view is consistent with the *correct* interpretation of stellar aberration. To this end, one should look at refs.[11, 12]. In particular, in Phipps’s paper [12], that starts from the original De Sitter’s remark that double star systems with high relative velocities show the same aberration angle, it is clearly pointed out that the experimental value $\theta \sim 20.5''$ with $\tan \theta \sim \frac{v}{c}$, defines a velocity $v \sim 30$ km/sec that is *not* the relative velocity between the observer and the emitting source. Rather, v is associated with the motion of the observer, in our case the earth, in the gravitational field of the sun. The same conclusion is suggested by Synge [13] who states that for observational purposes, the two frames S and S’ (connected by

the Lorentz transformation with velocity v) "...consist of the earth himself at two different positions in its orbit around the sun". This implies that an hypothetical observer placed on Pluto, would observe an aberration angle $\theta \sim 3.3''$ and that another hypothetical observer placed on the Sun would observe no aberration at all. This last conclusion is indeed consistent with Bergman's treatment [14] where the aberration angle relates "...the direction of the incoming light with respect to two frames of reference, that of the sun and that of the earth".

Therefore, since De Sitter's remark dates back to 1913, one can understand why the idea of General Relativity as a description of the deformations of a peculiar, aethereal medium was seriously considered by Einstein in the period 1916-1924. This 'resurrection of aether' in Einstein's mind is confirmed by a considerable amount of published and unpublished manuscripts reported by Kostro [15], including the famous Leyden lecture. According to this original picture, the inclusion of gravity, and therefore the transition from Special to General Relativity, could be understood by replacing the aether of classical physics with a 'sublimated' aether whose constituents do not follow definite space-time trajectories: today we would say 'that have a quantum behaviour'. However, independently of quantum mechanics, looking at ref.[15], one will discover that there might have been several reasons, even quite unrelated to physics, why the 'sublimated' aether was not exploited by Einstein in more detail.

At the same time, just the quantum phenomena might induce to a change of perspective in the approach to 'geometrize' gravity. This, historically, started from experimental observations, namely the universality of free fall. However, other experiments show that quantum mechanical wave functions are very different from the geometrical objects of classical tensor calculus [16]. In fact, one can explicitly prove [17] the 'strong' equivalence between a gravitational field and an accelerated frame. The price to pay is to admit mass-dependent phase transformations of the wave functions. Therefore, the possibility of re-absorbing the effects of gravity into a universal change of space-time might simply reflect the net cancellation of the quantum interference effects in the classical limit. This might be another indication for General Relativity being an effective theory which averages over distances that are much larger than the typical atomic sizes.

In this spirit, we shall start in Sect.2 from the basic properties of curved space-time where, for long wavelengths, the excitations of our medium can be described in terms of a single scalar function $\sigma(x)$. This is *experimentally* known to be the Newton potential. Its possible interpretation in terms of collective density fluctuations of the Higgs condensate will be proposed in Sect.3. Finally, Sect.4 will contain our summary and a discussion of some general consequences of our approach.

2 Curved space-time and a ‘medium’ with a scalar field

Describing the possible excitation states of a superfluid medium is an extremely difficult task. In this section, we shall adopt a minimal point of view, namely trying to re-absorb its properties into a single scalar function

$$\Phi(x) \equiv e^{-\sigma(x)} \langle \Phi \rangle \quad (2)$$

that deviates from its equilibrium constant value corresponding to $\sigma = 0$. As anticipated in the Introduction, this type of description should work in the hydrodynamical regime, when the wavelengths associated with the fluctuations are much larger than the mean free path for the elementary constituents.

As a possible form of interaction between the fluctuations of the medium and ordinary matter, we shall adopt the simple Higgs model, namely a coupling to the particle mass with the replacement

$$m_o \rightarrow m_o \frac{\Phi(x)}{\langle \Phi \rangle} = m_o e^{-\sigma(x)} \quad (3)$$

As anticipated, we shall also assume that $\sigma(x)$ is a very slowly varying function such that one can completely neglect its variation over distances that are comparable with the atomic size. In this situation, the effect of a non-zero σ on the energy levels of a hydrogen-like atom simply amounts to a re-definition of the electron mass with an average constant value $m = m_o e^{-\sigma}$ in the Dirac Hamiltonian

$$H_D = \alpha \cdot \mathbf{p} + \beta m - \frac{Ze^2}{r} \quad (4)$$

This changes the energy levels and the frequencies $\omega_o \rightarrow \omega_o e^{-\sigma}$. Therefore, the natural period of an atomic clock $T = \frac{2\pi}{\omega}$ is changed, $T = T_o e^{\sigma}$, with respect to the value $T_o = \frac{2\pi}{\omega_o}$ associated with $\sigma = 0$. Analogously, the Bohr radius $r_B = \frac{\hbar}{Ze^2 m_o}$ is changed into $r_B e^{\sigma}$ thus producing a symmetric re-scaling of the length of the rods. Since all masses are affected in the same way, and the units of length and time scale as inverse masses, the overall effect is equivalent to a conformal re-scaling of the metric tensor

$$g_{\mu\nu}(x) = e^{2\sigma(x)} \eta_{\mu\nu} \quad (5)$$

where $\eta_{\mu\nu}$ denotes the Minkowski metric.

However, the idea of $\sigma(x)$ as the fluctuation of a (‘non-dispersive’) medium, suggests another physical effect: the introduction of a refractive index. In this case, before the conformal re-scaling, the Minkowski metric would be replaced by

$$\hat{\eta}_{\mu\nu} \equiv \left(\frac{1}{N^2}, -1, -1, -1 \right) \quad (6)$$

with a refractive index $\mathcal{N} = \mathcal{N}(\sigma)$ and a normalization such that $\mathcal{N} = 1$ when $\sigma = 0$ (when no confusion can arise, we shall set to unity the Newton constant G_N and the speed of light in the ‘vacuum’ c).

Thus we obtain the metric structure

$$\hat{g}_{\mu\nu} \equiv (\frac{e^{2\sigma}}{\mathcal{N}^2}, -e^{2\sigma}, -e^{2\sigma}, -e^{2\sigma}) \quad (7)$$

that re-absorbs the local, isotropic modifications of Minkowski space into its basic ingredients: the value of the speed of light and the space-time units. Equivalently, the same metric structure can be interpreted as arising from *separate* local changes of the space and time units. In fact, such a transformation is known to represent one of the many possible ways, perhaps the most fundamental, to introduce the concept of curvature [18, 19, 20]. In particular, there exist definitions of units, depending on a scalar field, for which a general curved space-time becomes flat, all the Riemannian invariants being zero [21].

The idea of introducing a refractive index is very natural (see e.g. [22]) at least when comparing with any known medium with definite physical properties. For instance, when considering, as a possible model, the non-trivial vacuum of an underlying scalar quantum field theory, we get the simple picture of a condensate of spinless quanta. If these are treated as ‘hard-spheres’, the physical properties of the vacuum would depend on their scattering length and particle density. On this basis, Lenz [23] first showed that such a system behaves like a medium with a refractive index.

Without attempting a microscopic derivation, we can try to deduce a possible form of $\mathcal{N}(\sigma)$ using some general arguments. To this end, we first observe that, for a time-independent $\sigma(x)$, our metric Eq.(7) is just a different way to write the general isotropic metric

$$\hat{g}_{\mu\nu} \equiv (A, -B, -B, -B) \quad (8)$$

Now, one may ask when the local light velocity $c(x, y, z) \equiv \sqrt{\frac{A}{B}}$, defined as a ‘particle’ velocity from the condition $ds^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = 0$, agrees with the curved-space equivalent of the phase velocity of light pulses. These are solutions of the D’Alembert wave equation with the metric $(A, -B, -B, -B)$ [26]

$$\frac{1}{A} \frac{\partial^2 F}{\partial t^2} - \frac{1}{B} (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) F - \frac{1}{\sqrt{AB^3}} (\nabla \sqrt{AB}) \cdot (\nabla F) = 0 \quad (9)$$

so that, by introducing the 3-vector $\mathbf{g} \equiv \sqrt{\frac{A}{B^3}} (\nabla \sqrt{AB})$ we obtain

$$\frac{\partial^2 F}{\partial t^2} = \frac{A}{B} \Delta F + \mathbf{g} \cdot (\nabla F) \quad (10)$$

By identifying $\frac{1}{F} \frac{\partial^2 F}{\partial t^2}$ as the equivalent of $-\omega^2$ and $\frac{1}{F} \Delta F$ as the corresponding of $-k^2$, we find that particle velocity and phase velocity $c_{\text{ph}} \equiv \frac{\omega}{k}$ agree with each other only when $\mathbf{g} = 0$, i.e. when AB is a constant. This product can be fixed to unity with flat-space boundary conditions at infinity and, therefore, the resulting value

$$\mathcal{N} = e^{2\sigma} \quad (11)$$

can be considered a consistency requirement on our medium to preserve the observed particle-wave duality which is intrinsic in the nature of light. On the other hand, if $\mathcal{N} \neq e^{2\sigma}$, we should specify the operative definition used for the local speed of light: a) the time difference for a light pulse to go forth and back between two infinitesimally close objects at relative rest, b) the value obtained combining frequency and wavelength of a given radiation source,...

With this choice, Eq.(7) becomes

$$\hat{g}_{\mu\nu} \equiv (e^{-2\sigma}, -e^{2\sigma}, -e^{2\sigma}, -e^{2\sigma}) \quad (12)$$

and could be used for a space-time description in a medium specified by a given scalar field $\sigma(x)$.

Alternatively, if we assume that our $\sigma(x) \neq 0$ is equivalent to a gravitational field, we can compare the metric structure Eq.(12) with the experiments in the known weak gravitational fields where the gravitational potential does not change appreciably over distances, say, of a few millimeters. In this case, we obtain the experimental result that σ is just the Newton potential, i.e. (for a centrally symmetric field)

$$\sigma_{\text{exp}} = \frac{M}{r} \quad (13)$$

As anticipated, in this description, the space-time curvature is equivalent to suitable local re-definitions of the space and time units. Thus, for instance, the gravitational red-shift is explained through the behaviour of clocks in the gravitational field whereas the energy of the propagating photon does *not* change with the height [24] and there is no need to introduce the concept of an effective gravitational mass for the propagating photons. Analogously, the deflection of light can be explained by converting the units of time and length that define the local speed of light into those that are used for the plotted speed [25].

Let us now compare our ‘medium’ with General Relativity. To this end, we shall study the class of metrics that are solutions of the following field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \lambda(\sigma_\mu \sigma_\nu - \frac{1}{2} g_{\mu\nu} \sigma^\alpha \sigma_\alpha) \quad (14)$$

These can be considered Einstein equations in the presence of an energy-momentum tensor for a scalar field $\sigma(x)$, for various values of the parameter λ . For a centrally symmetric field $\sigma = \sigma(r)$, Tupper [27] has shown that *all* solutions of Eqs.(14) are consistent with the three classical weak-field tests and the Shapiro planetary radar reflection experiment. Depending on the value of the parameter λ one gets the Schwarzschild metric, for $\lambda = 0$, or the Yilmaz metric [28], for $\lambda = 2$. For any λ , the solutions are conformal transformations of solutions of the Brans-Dicke theory [29].

However, only for $\lambda = 2$ the resulting metric tensor depends *parametrically* on σ . In this case, Eqs.(14) become algebraic *identities* consistently with the point of view Eq.(1) that the metric tensor is an auxiliary kinematical quantity depending parametrically on more fundamental fields. In this case, for $\lambda = 2$, where $\sigma = \frac{M}{r}$ represents the Newton potential, the Yilmaz metric in isotropic form ('Y'=Yilmaz)

$$g_{\mu\nu}^Y \equiv (e^{-\frac{2M}{r}}, -e^{\frac{2M}{r}}, -e^{\frac{2M}{r}}, -e^{\frac{2M}{r}}) \quad (15)$$

is formally identical to Eq.(12).

Notice that the idea of the Newton potential as a fundamental scalar field is *not* restricted to flat space but has a meaning also in curved space-time. Clearly, adopting the point of view of a parametric dependence of the metric tensor is equivalent to say that the 'right' choice among all possible values of λ , is just $\lambda = 2$, i.e. the old Yilmaz theory. Finally, having an independent argument for $\sigma(x)$ being the Newton potential, we could describe the relevant space-time properties without solving any field equation.

The choice $\lambda = 2$ is also suggested by the Yilmaz remark [30] that the metric Eq.(12), differently from all other metrics, is a solution of Einstein field equations, even for a gravitational many-body system where $T_{\mu\nu} \equiv (\rho, 0, 0, 0)$ and

$$\rho(\mathbf{x}) = -4\pi \sum_n M_n \delta^3(\mathbf{x} - \mathbf{x}_n) \quad (16)$$

Indeed, when replacing the Newton potential

$$\sigma(\mathbf{x}) = \sum_n \frac{M_n}{|\mathbf{x} - \mathbf{x}_n|} \quad (17)$$

in the metric Eq.(12), the field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2(\sigma_\mu\sigma_\nu - \frac{1}{2}g_{\mu\nu}\sigma^\alpha\sigma_\alpha) + 2T_{\mu\nu} \quad (18)$$

become algebraic identities. Therefore, if one wants to compare with real many-body gravitational systems, Eq.(12) represents a very convenient starting point for any time-dependent

approximation. Indeed, for slow motions, the situation is similar to the conventional adiabatic Born-Oppenheimer approximation where one approaches the time-dependent problem by expanding in the eigenfunctions of a static 2-center, 3-center,... n-center hamiltonian. Actually, by inspection of the second-order terms, Yilmaz claims [31] that the standard Einstein-Hoffman-Infeld metric, used for a description of classical motions in the solar system, represent a time-dependent improvement on the static solution of Eq.(18) with $\sigma(x)$ as in Eq.(17). Namely, the scalar field would be implicitly taken into account. Without committing ourselves to this particular aspect, we shall return, however, to a comparison with the Schwarzschild metric in the conclusions.

Before concluding, we observe that the metric structure Eq.(12) was also obtained by Dicke [32] in a stimulating remake of Lorentz's electromagnetic aether. This is based on the simultaneous replacements of the particle mass

$$m_o \rightarrow m_o f(\epsilon, \mu) \quad (19)$$

and of the light velocity

$$c^2 \rightarrow \frac{c^2}{\epsilon \mu} \quad (20)$$

where ϵ and μ are respectively the space-time dependent dielectric function and magnetic permeability of the aether. Consistency with the experimental results (the Eötvös experiment, velocity independence of the electric charge,...) requires $\epsilon = \mu$ leading to the effective metric structure ('LD'=Lorentz-Dicke)

$$g_{\mu\nu}^{\text{LD}} \equiv \left(\frac{f^2}{\epsilon^4}, -\frac{f^2}{\epsilon^2}, -\frac{f^2}{\epsilon^2}, -\frac{f^2}{\epsilon^2} \right) \quad (21)$$

Finally, a comparison with the classical tests in a weak gravitational field gives

$$f^2 = \epsilon^3 \quad (22)$$

and

$$\epsilon = 1 + 2\frac{M}{r} + \dots = e^{2\sigma} \quad (23)$$

so that Eq.(21) reduces to Eq.(12).

This last argument confirms the very general nature of Eq.(12), σ being the Newton potential. This structure reproduces, in the experimentally available, slowly varying gravitational fields, the known classical space-time effects, and is in agreement with our starting point Eq.(1). As anticipated in the Introduction, this does not mean that 'all' possible gravitational phenomena (including those yet undiscovered) can be described in terms of Eq.(12).

For instance, as we shall comment in Sect.4, one can try to identify transverse vibrations in the scalar condensate. If these were substantially excited, the required space-time description might no longer fit with the metric Eq.(12). However, for the time being, by restricting to the experimentally available gravitational fields, the known space-time effects can also be described in terms of our medium by introducing simple modifications of the flat Minkowski metric depending on a scalar field $\sigma(x)$. This type of approach does not require to solve any field equations but requires to understand *why* $\sigma(x)$ is the Newton potential.

3 Long-wavelength excitations of the Higgs condensate

3.1 Let us now investigate a possible physical origin of σ . This requires a physical framework where we can understand its identification with the Newton potential, i.e. a model field equation that reduces to the Poisson equation in the static limit. To start with, let us first write σ in its correct dimensionless form. For instance, for the centrally symmetric case the correct relation is

$$\sigma(r) = \frac{G_N M}{c^2 r} \quad (24)$$

where $G_N = 6.67 \cdot 10^{-8} \text{cm}^3 \text{sec}^{-2} \text{gr}^{-1}$, $c = 2.9979 \cdot 10^{10} \text{cm sec}^{-1}$, M is in grams and r in centimeters. Therefore, σ is a solution of the Poisson equation

$$\Delta \sigma = \frac{G_N}{c^2} \rho \quad (25)$$

with

$$\rho = -4\pi M \delta^{(3)}(\mathbf{r}) \quad (26)$$

Now, since σ is a scalar, the simplest way to recover Eq.(25), is to consider a D'Alembert wave equation with some source $S(x)$ that, in the static limit, becomes proportional to the mass density ρ . This leads, in general, to introduce a parameter η and the equation

$$(\eta \Delta - \frac{\partial^2}{c^2 \partial t^2}) \sigma = -4\pi \frac{\tilde{G}_N}{c^2} S(x) \quad (27)$$

with

$$\tilde{G}_N = \eta G_N \quad (28)$$

The introduction of η is useful to transform Eq.(27) into

$$(\Delta - \frac{\partial^2}{c_s^2 \partial t^2}) \sigma = -4\pi \frac{G_N}{c^2} S(x) \quad (29)$$

In fact, if $\sigma(x)$ were associated with a medium, Eq.(29) would describe density fluctuations propagating with a squared ‘sound velocity’

$$c_s^2 = \eta c^2 \quad (30)$$

Let us now analyze the implications of this toy-model. Using Eq.(27), we could try to relate the scalar field to a typical particle physics scale \tilde{G}_N . In this case, however, the same re-scaling η , relating \tilde{G}_N to G_N affects the relation between c_s^2 and c^2 . Therefore, assuming a typical particle scale for \tilde{G}_N , say the Fermi constant G_F , would require density fluctuations propagating at the fantastically high speed $c_s \sim 4 \cdot 10^{16} c$. Is this conceivable? For definiteness, we shall explore a condensate of spinless quanta to check whether the idea $c_s \sim 4 \cdot 10^{16} c$ is plausible, or not.

3.2 Before addressing the physical properties of a scalar condensate, however, one premise is in order. For a spontaneously broken one-component $\lambda\Phi^4$ theory, i.e. leaving aside the Goldstone bosons, the particle content of the broken phase is usually represented as a single massive field, the (singlet) Higgs boson. Although there is no rigorous proof [33], the energy spectrum of the broken phase is believed to tend to a non-zero value, $\tilde{E}(\mathbf{p}) \rightarrow M_h$, when $\mathbf{p} \rightarrow 0$ so that the non-zero quantity $\tilde{E}(0) = M_h$ should give rise to an exponential decay $\sim e^{-\tilde{E}(0)T}$ of the connected Euclidean propagator. This is equivalent to require that the Fourier transform of the connected Euclidean propagator tend to a finite limit, $G(p) \rightarrow \frac{1}{M_h^2}$ when the 4-momentum $p \rightarrow 0$, with the mass squared M_h^2 being related to the quadratic shape of a semi-classical, non-convex effective potential $V_{\text{NC}}(\phi)$ (‘NC’=non-convex) at its non-trivial absolute minima, say $\phi = \pm v$.

However, in a spontaneously broken phase, it can be shown [34, 35] that $G(0)$ is a *two-valued* function that includes the case $G^{-1}(0) = 0$ as in a gap-less theory. This effect cannot be discovered in a conventional perturbative calculation where the zero-mode of the scalar field is ‘frozen’ at one of the two minima of $V_{\text{NC}}(\phi)$. To this end, one has to go beyond the simplest approximation where the ‘Higgs condensate’ is treated as a classical c-number field. Namely, one has either to perform the last functional integration over the zero-mode of the scalar field [34] or first re-sum the one-particle reducible, zero-momentum tadpole graphs in the constant background field ϕ [35], by taking the limit $\phi \rightarrow \pm v$ at the end of the calculation. The tadpole graphs, in fact, are connected to the other parts of the diagrams through zero-momentum propagators and can be considered a manifestation of the quantum nature of the scalar condensate. In both cases, one finds *two* possible solutions for the inverse

zero-4-momentum connected propagator at $\phi = \pm v$: a) $G_a^{-1}(0) = M_h^2$ and b) $G_b^{-1}(0) = 0$.

Let us first review the results of ref.[34]. The starting point is the separation of the scalar field into a constant background and a shifted fluctuation field, namely

$$\Phi(x) = \phi + h(x) \quad (31)$$

In order Eq.(31) to be unambiguous, ϕ denotes the spatial average in a large 4-volume Ω

$$\phi = \frac{1}{\Omega} \int d^4x \Phi(x) \quad (32)$$

and the limit $\Omega \rightarrow \infty$ has to be taken at the end.

In this way, the full functional measure can be expressed as

$$\int [d\Phi(x)] \dots = \int_{-\infty}^{+\infty} d\phi \int [dh(x)] \dots \quad (33)$$

and the functional integration on the r.h.s. of Eq.(33) is over all quantum modes with 4-momentum $p \neq 0$.

After integrating out all non-zero quantum modes, the generating functional in the presence of a space-time constant source J is given by

$$Z(J) = \int_{-\infty}^{+\infty} d\phi \exp[-\Omega(V_{\text{NC}}(\phi) - J\phi)] \quad (34)$$

to any finite order in the loop expansion. Finally, by introducing the generating functional for connected Green's functions $w(J)$ through

$$\Omega w(J) = \ln \frac{Z(J)}{Z(0)} \quad (35)$$

one can compute the field expectation value

$$\varphi(J) = \frac{dw}{dJ} \quad (36)$$

and the $p_\mu = 0$ propagator

$$G_J(0) = \frac{d^2w}{dJ^2} \quad (37)$$

In this framework, spontaneous symmetry breaking corresponds to non-zero values of Eq.(36) in the double limit $J \rightarrow \pm 0$ and $\Omega \rightarrow \infty$.

Now, by denoting $\pm v$ the absolute minima of V_{NC} and

$$M_h^2 \equiv \frac{d^2V_{\text{NC}}}{d\phi^2} \quad (38)$$

its quadratic shape at these extrema, one usually assumes

$$\lim_{\Omega \rightarrow \infty} \lim_{J \rightarrow \pm 0} \varphi(J) = \pm v \quad (39)$$

and

$$\lim_{\Omega \rightarrow \infty} \lim_{J \rightarrow \pm 0} G_J(0) = \frac{1}{M_h^2} \quad (40)$$

In this case, the excitations in the broken phase would be massive particles (the conventional Higgs bosons) whose mass M_h is determined by the positive curvature of V_{NC} at its absolute minima. However, at $\varphi = \pm v$, besides the value $\frac{1}{M_h^2}$, one also finds [34]

$$\lim_{\Omega \rightarrow \infty} \lim_{J \rightarrow \pm 0} G_J(0) = +\infty \quad (41)$$

a result that has no counterpart in perturbation theory.

Analogously, after including the one-particle reducible, zero-momentum tadpole graphs, the formal power series for the exact inverse zero-momentum propagator can be expressed as [35]

$$G^{-1}(0) = \left. \frac{d^2 V_{\text{NC}}}{d\phi^2} \right|_{\hat{\phi}=\phi(1-\tau)} \quad (42)$$

with

$$\tau \equiv \tilde{T}(\phi^2)G(0) \quad (43)$$

where the tadpole function $\tilde{T}(\phi^2)$ vanishes at $\phi = \pm v$. As a result, after including tadpole graphs to all orders, one finds multiple solutions for the zero-4-momentum propagator that again can differ from (38) even when $\phi \rightarrow \pm v$.

In fact, Eq.(42) provides a regular solution $G_a^{-1}(0) = M_h^2$ for which

$$\lim_{\phi \rightarrow \pm v} \tau = \bar{\tau} = 0 \quad (44)$$

and a singular solution $G_b^{-1}(0) = 0$ such that

$$\lim_{\phi \rightarrow \pm v} \tau = \bar{\tau} \neq 0 \quad (45)$$

As an example, let us consider the situation of the tree-level approximation where $V_{\text{NC}} \equiv V_{\text{tree}}$ with

$$V_{\text{tree}} = \frac{1}{2}r\phi^2 + \frac{\lambda}{4!}\phi^4 \quad (46)$$

an approximation which is equivalent to re-summing tree-level tadpole graphs to all orders (i.e. no loop diagrams). In this case the regular solution is $G_a^{-1}(0) = \frac{\lambda v^2}{3}$, while the singular solution is

$$\lim_{\phi \rightarrow \pm v} G_b^{-1}(0) = \frac{\lambda v^2}{2} [\bar{\tau}^2 - 2\bar{\tau} + \frac{2}{3}] = 0 \quad (47)$$

which implies limiting values $\bar{\tau} = 1 \pm \frac{1}{\sqrt{3}}$.

In general, beyond the tree-approximation, finding the singular solution $G_b^{-1}(0) = 0$ at $\phi = \pm v$ is equivalent to determine that value of $\hat{\phi}^2 \equiv v^2(1 - \bar{\tau})^2$ where $\frac{d^2 V_{\text{NC}}}{d\phi^2} = 0$. For instance, in the case of the Coleman-Weinberg effective potential

$$V_{\text{NC}}(\phi) = \frac{\lambda^2 \phi^4}{256\pi^2} \left(\ln \frac{\phi^2}{v^2} - \frac{1}{2} \right) \quad (48)$$

the required values are $\bar{\tau} = 1 \pm e^{-1/3}$. In principle, such solutions exist in any approximation to V_{NC} due to the very general properties of the shape of a non-convex effective potential.

The b-type of solution corresponds to processes where absorbing (or emitting) a very small 3-momentum $\mathbf{p} \rightarrow 0$ does not cost a finite energy. This situation is well known in a condensed medium where a very small 3-momentum can be coherently distributed among a large number of elementary constituents (the hydrodynamical regime mentioned in the Introduction). Therefore, as for ^4He with phonons and rotons, in a spontaneously broken phase, there are actually *two* possible types of excitations with the same quantum numbers but different energies when $\mathbf{p} \rightarrow 0$: a massive one, with $\tilde{E}_a(\mathbf{p}) \rightarrow M_h$, and a gap-less one with $\tilde{E}_b(\mathbf{p}) \rightarrow 0$. They can both propagate (and interfere) in the broken phase. However, the gap-less excitation would dominate the exponential decay $\sim e^{-\tilde{E}_b(\mathbf{p})T}$ of the connected euclidean correlator for $\mathbf{p} \rightarrow 0$ so that the massive mode becomes unphysical in the infrared region. Therefore, differently from the simplest perturbative indications, in a (one-component) spontaneously broken $\lambda\Phi^4$ theory there would be no energy-gap associated with the ‘Higgs mass’ M_h , as for a genuine massive single-particle theory where the massive spectrum

$$\tilde{E}(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M_h^2} \quad (49)$$

remains true for $\mathbf{p} \rightarrow 0$. Rather, the infrared region would be dominated by the gap-less mode.

We observe that ref.[5], although providing the physical mechanism for the phion-condensation phenomenon, did not show any evidence for the existence of the gap-less branch. This is due to the particular approximation, where the creation and annihilation operators $a_{\mathbf{p}=0}$, $a_{\mathbf{p}=0}^\dagger$ for the elementary quanta in the $\mathbf{p} = 0$ mode were simply replaced by the c-number \sqrt{N} .

This choice is the second-quantization analog of ‘freezing’ $\phi = \pm v$ without performing the functional integration as in ref.[34] or without first re-summing the zero-momentum tadpole graphs as in ref.[35]. In this case, the only solution is $G^{-1}(0) = M_h^2$.

Let us now address the physical properties of the gap-less branch using the standard treatment of the long-wavelength excitations in quantum liquids.

3.3 The quantum hydrodynamical picture of zero-temperature Bose systems has a basic universal feature: the lowest excitations are *phonons*. ”These are excited states of compression generated by small displacements of the elementary atoms with a resulting infinitesimal change of their density” [36]. In other words, in a quantum Bose liquid”... elementary excitations with small momenta \mathbf{p} (wavelengths large compared with distances between atoms) correspond to ordinary hydrodynamic sound waves, i.e. are phonons. This means that the energy of such quasi-particles is a linear function of their momentum” [37]. At zero temperature, they behave as non-interacting particles so that their energy spectrum,

$$\tilde{E}(\mathbf{p}) \equiv c_s |\mathbf{p}| \quad (50)$$

(c_s being the sound velocity) is virtually exact for $\mathbf{p} \rightarrow 0$. The only restriction is due to the limitation in the \mathbf{p} values for which Eq.(50) applies. These extend up to a maximum momentum $|\mathbf{p}|_{\max} \equiv \delta$ which is considerably smaller than the inverse mean free path R_{mfp} for the elementary constituents

Therefore, in a zero-temperature scalar condensate, and in the limit $\mathbf{p} \rightarrow 0$, quantum hydrodynamics predicts long-wavelength excitations to be density fluctuations. In this approach, the phion condensate undergoes small oscillations where the local density $n(\mathbf{r})$ is very close to its equilibrium value n_o , i.e. $|n(\mathbf{r}) - n_o| \ll n_o$. Also, the current flow is such that $|n\mathbf{v}| \ll n_o c_s$ and the velocity field \mathbf{v} describes a potential flow

$$\mathbf{v} = \nabla \Phi \quad (51)$$

so that one can replace $\nabla(n\mathbf{v}) \sim n_o \Delta \Phi$, obtaining the ‘Poisson regime’.

Thus, we understand the gap-less mode discovered in refs.[34, 35] as being due to the propagation of phonons in the phion condensate and we can start to figure out the possible values of c_s . To this end, we first observe that the idea of a ‘quantum aether’ was considered by Dirac [38]. In his approach, there is a time-like vector field v_μ , with $v_\mu v^\mu = 1$, that, in a classical picture, would correspond to an aether velocity field. For a quantum theory, rather than considering operatorial relations, one may introduce [38] a wave function for the aether

and require its modulus to be independent of v_μ , as with plane-waves in quantum mechanics. Let us exploit some consequence of this idea and consider the usual Klein-Gordon equation for particles of physical mass $m_\Phi^2 > 0$

$$\left(\frac{\partial^2}{\partial t^2} - \Delta\right)\Phi = -m_\Phi^2\Phi \quad (52)$$

We shall restrict to solutions of the form $\Phi = \Phi(w)$ where $w = v_\mu x^\mu$ is the Lorentz-invariant variable defined at each space-time point in a quantum aether with a given constant $v_\mu \equiv (v_o, \mathbf{v})$. Periodic solutions have the form $\hat{\Phi} = a \cos(m_\Phi w) + b \sin(m_\Phi w)$ and would usually be interpreted as particles with 4-momentum $p_\mu = m_\Phi v_\mu$ and mass-shell condition $p^2 = m_\Phi^2$.

However, let us slightly change perspective and interpret the same Klein-Gordon equation in a statistical sense, i.e. as the equation governing the collective wave-function of a ‘Madelung fluid’ [39] of phions moving with velocity \mathbf{v} . In this case, the *same* periodic solutions, satisfying the wave-equation

$$(c_s^2 \Delta - \frac{\partial^2}{\partial t^2})\hat{\Phi}(w) = 0 \quad (53)$$

with

$$c_s^2 \equiv 1 + \frac{1}{|\mathbf{v}|^2} \quad , \quad (54)$$

would naturally be interpreted as *density waves*, in agreement with the phonon energy spectrum Eq.(50) expected on the base of Landau’s hydrodynamical picture.

Notice that the phonon-like excitations of the phion condensate propagate with a superluminal velocity $c_s > 1$. Therefore, near the aether rest-frame, i.e. where $|v_o| \sim 1$ and $|\mathbf{v}|^2 \rightarrow 0$, these waves would be seen to propagate with a very large speed $c_s \rightarrow \infty$. In addition, with the same space-time units, one can introduce a space-like vector s_μ such that $s^\mu s_\mu = -1$ and $s_\mu v^\mu = 0$ as for a genuine vacuum angular momentum. By considering the same Klein-Gordon equation (52) for a function $\chi = \chi(\xi)$, ξ being the other Lorentz-invariant combination $\xi = s_\mu x^\mu$, the equivalent waves associated with χ propagate with a subluminal velocity $c_s < 1$.

Obviously, analogous results hold in a reversed form. For instance, let us start from any superluminal wave equation with $c_s^2 \equiv 1 + \beta^2 > 1$

$$(c_s^2 \Delta - \frac{\partial^2}{\partial t^2})\varphi = 0 \quad (55)$$

and consider its plane wave solutions $\varphi = \varphi(w)$, with $w = \sqrt{1 + \frac{1}{\beta^2}}t - \frac{1}{\beta}\mathbf{n} \cdot \mathbf{r}$, depending on an arbitrary spatial direction \mathbf{n} with $\mathbf{n} \cdot \mathbf{n} = 1$. The periodic solutions $\hat{\varphi}(\tau)$, such that $\frac{d^2 \hat{\varphi}}{d\tau^2} = -\hat{\varphi}$,

depend on a dimensionless parameter $\tau \equiv \alpha w$, α being a physical mass scale introduced to define the periodicity condition in some units. They are solutions of a Klein-Gordon equation

$$(\frac{\partial^2}{\partial t^2} - \Delta)\hat{\varphi} = -\alpha^2 \hat{\varphi} \quad (56)$$

with a physical mass-squared $\alpha^2 > 0$. In this case, the Lorentz-covariant equation emerges when the dependence on the hidden mass scale α is made explicit.

The same idea that density fluctuations in a Higgs condensate propagate with a velocity c_s which is *infinitely* larger than the speed of light is also suggested by a semi-classical argument due to Stevenson [40] that we shall briefly report. Stevenson's argument starts from a perfect-fluid treatment of the Higgs condensate in its rest frame. The perfect-fluid approximation is frequently used in the literature, for instance in cosmology. If there is a Higgs condensate around, this approximation may also apply to it. Now, in this approximation, energy-momentum conservation is equivalent to wave propagation with a squared velocity given by

$$c_s^2 = c^2 (\frac{\partial \mathcal{P}}{\partial \mathcal{E}}) \quad (57)$$

where \mathcal{P} is the pressure and \mathcal{E} the energy density. Introducing the condensate density n , and using the energy-pressure relation

$$\mathcal{P} = -\mathcal{E} + n \frac{\partial \mathcal{E}}{\partial n} \quad (58)$$

we obtain

$$c_s^2 = c^2 (\frac{\partial \mathcal{P}}{\partial n}) (\frac{\partial \mathcal{E}}{\partial n})^{-1} = c^2 (n \frac{\partial^2 \mathcal{E}}{\partial n^2}) (\frac{\partial \mathcal{E}}{\partial n})^{-1} \quad (59)$$

For a non-relativistic Bose condensate of neutral particles with mass m and scattering length a , where $\frac{na\hbar^2}{m^2 c^2} \ll 1$ (in this case we also explicitly introduce \hbar) one finds

$$\mathcal{E} = nmc^2 + n^2 \frac{2\pi a \hbar^2}{m} \quad (60)$$

so that the sound velocity is

$$c_s^2 = \frac{4\pi na \hbar^2}{m^2} \quad (61)$$

Eq.(61) agrees with the sound velocity obtained from the *microscopic* [41] derivation of the Bogolubov spectrum in a dilute hard-sphere Bose gas.

On the other hand, when describing the occurrence of spontaneous symmetry breaking, where the scalar quanta are pions with mass m_Φ , there are now additional terms in Eq.(60).

These are such that the scalar condensate is spontaneously generated from the ‘empty’ vacuum where $n = 0$ for that particular equilibrium phion density where [5]

$$\frac{\partial \mathcal{E}}{\partial n} = 0 \quad (62)$$

Therefore, in this approximation, approaching the equilibrium density one finds

$$c_s^2 \rightarrow \infty \quad (63)$$

implying, again, that long-wavelength density fluctuations would propagate instantaneously in the spontaneously broken vacuum.

As Stevenson points out [40], Eq.(63) neglects all possible corrections to the perfect-fluid approximation, just as Eq.(52) neglects the effect of a weak phion self-coupling λ . These phion-phion interactions introduce collisional effects associated with a finite mean free path $R_{\text{mfp}} \sim \frac{1}{na^2}$, n being the phion number density and a their S-wave scattering length [42]. Due to its finite value, density waves will propagate at a large but finite speed c_s such that $c_s \rightarrow \infty$ when $\lambda \rightarrow 0$. Also their wavelengths will be larger than R_{mfp} [40] in agreement with the hydrodynamical restriction of the energy spectrum Eq.(50) to momenta $|\mathbf{p}| < R_{\text{mfp}}^{-1}$.

Finally, the result $c_s \rightarrow \infty$ is also suggested by a third argument, if one takes into account the approximate nature of locality in cutoff-dependent quantum field theories. In this picture, the elementary quanta are treated as ‘hard spheres’, as for the molecules of ordinary matter. Thus, the notion of the vacuum as a ‘condensate’ acquires an intuitive physical meaning. For the same reason, however, the simple idea that deviations from Lorentz-covariance take only place at the cutoff scale may be incorrect. In fact, a hard-sphere radius is known, from the origin of Special Relativity [43], to imply a superluminal propagation within the sphere boundary. Now, in the perturbative empty vacuum state (with no condensed quanta) such superluminal propagation is restricted to very short wavelengths, smaller than the inverse ultraviolet cutoff. However, in the condensed vacuum, the hard spheres can ‘touch’ each other so that the actual propagation of density fluctuations in a hard-sphere system might take place at a superluminal speed. This is very close to our previous ‘dual’ picture of the Klein-Gordon equation. Although the individual particles are limited to a time-like motion, there are real physical collective excitations, in the form of density waves [44] that propagate at a superluminal speed. This provides a definite model [44] for the non-local nature of the quantum potential [39] and, as such, a possible answer to the delicate questions raised by Bell’s inequality.

In this sense, hard-sphere condensation, as a model of the broken-symmetry vacuum in a cutoff theory, leads to what Volovik calls *reentrant violations of special relativity in the low-energy corner* [45]. This produces a simple physical picture in terms of the two different solutions [34, 35] for the zero-momentum propagator. In fact, let us consider an infinite, isotropical scalar condensate where both translational and rotational invariance is preserved. In this case, the possible *reentrant* violations will extend over a small shell of momenta, say $|\mathbf{p}| < \delta$, where the condensate excitation spectrum $\tilde{E} = \tilde{E}(|\mathbf{p}|)$ deviates from a Lorentz-covariant form. However, full Lorentz covariance has to be re-established in the local limit. Therefore, for a large but finite ultraviolet cutoff Λ , the scale δ is naturally infinitesimal in units of the scale associated with the Lorentz-covariant part of the energy spectrum, say M_h . By introducing dimensionless quantities, this means $\epsilon \equiv \frac{\delta}{M_h} \rightarrow 0$ when $t \equiv \frac{\Lambda}{M_h} \rightarrow \infty$ so that the continuum limit can equivalently be defined either as $t \rightarrow \infty$ or $\epsilon \rightarrow 0$. Notice that, formally, $\mathcal{O}(\frac{\delta}{M_h})$ vacuum-dependent corrections would represent $\mathcal{O}(\frac{M_h}{\Lambda})$ effects which are always neglected when discussing [46] how Lorentz-covariance emerges at scales much smaller than the ultraviolet cutoff. Therefore, although Lorentz-covariance is formally recovered in the local limit one finds, for large but finite Λ , infinitesimal deviations in an infinitesimal region of momenta.

Now, for our pion condensate, the quantum hydrodynamical analysis predicts the nature of the energy spectrum for $\mathbf{p} \rightarrow 0$. This has a very general nature, independently of any detail of the theory at the scale Λ , and corresponds to phonons as in Eq.(50) propagating with a sound velocity c_s , up to momenta $|\mathbf{p}| \sim \delta$ that start to be comparable with the inverse mean free path R_{mfp}^{-1} . There, we expect a transition to a Lorentz-covariant single-particle spectrum as in Eq.(49), so that

$$\sqrt{\delta^2 + M_h^2} \sim c_s \delta \quad (64)$$

or

$$c_s \sim \frac{M_h}{\delta} = \mathcal{O}(\frac{1}{\epsilon}) \quad (65)$$

This confirms that c_s , potentially, is an infinitely large quantity that diverges in the continuum limit. In fact, for a fixed mass scale M_h , the limit $t \rightarrow \infty$ can be replaced, equivalently, by $\epsilon \rightarrow 0$ or $c_s \rightarrow \infty$.

Due to this result, the massive branch will become predominant at momenta that are higher than δ . In fact, in this region, the collective excitations become unphysical since $c_s |\mathbf{p}|$ is now much larger than $\sqrt{\mathbf{p}^2 + M_h^2}$. Once more, this shows that the pion condensate, exhibiting a double-valued propagator [34, 35] and thus allowing for the propagation of two

different types of excitations, is very close to superfluid ^4He . There, the existence of two types of excitations was first deduced theoretically by Landau on the base of very general arguments [6]. According to that original idea, there would be phonons with energy $E_{\text{ph}}(\mathbf{p}) = v_s |\mathbf{p}|$ and rotons with energy $E_{\text{rot}}(\mathbf{p}) = \Delta + \frac{\mathbf{p}^2}{2\mu}$. Only *later*, it was experimentally discovered that there is a *single* energy spectrum $E(\mathbf{p})$ which is made up by a continuous matching of these two different parts. This unique spectrum agrees with the phonon branch for $\mathbf{p} \rightarrow 0$ and agrees with the roton branch at higher momenta.

We observe that, in the continuum limit $\frac{\Lambda}{M_h} \rightarrow \infty$, superluminal wave propagation is restricted to the region $|\mathbf{p}| < \delta$ with $\frac{\delta}{M_h} \rightarrow 0$. Therefore, in a strict local limit, it would be impossible to use these waves to form a sharp wave front and transfer informations with violations of causality [40]. These require values $\frac{d\tilde{E}}{d|\mathbf{p}|} > 1$ at large $|\mathbf{p}|$ that cannot occur due to the change of the energy spectrum from Eq.(50) to Eq.(49). On the other hand, for finite Λ , where δ is a finite scale, the same conclusion is not so obvious. In particular, the *reentrant* nature of the deviations from Lorentz covariance at small $|\mathbf{p}|$ is related to the genuine superluminal nature of the Fourier spectrum for $|\mathbf{p}| > \Lambda$ in cutoff theories. Therefore, there might be some differences with respect to other analyses [47] where the condition $\frac{d\tilde{E}}{d|\mathbf{p}|} > 1$ is shown not to be in conflict with causality. The nature of the problem, by itself, would deserve a dedicated effort. Here, we shall limit ourselves to these remarks and comment in the conclusions on possible superluminal effects in General Relativity, with and without a cosmological constant.

As anticipated, the limit $c_s \rightarrow \infty$ tries to simulate an exact Lorentz-covariant theory where the energy spectrum maintains its massive form down to $\mathbf{p} = 0$. Yet, this is not entirely true due to the subtleties associated with treating the zero-measure set $\mathbf{p} = 0$. This set, in fact, belongs to the range of Eq.(50) and therefore the right $c_s = \infty$ limit is always $\tilde{E}(\mathbf{p} = 0) = 0$ and not $\tilde{E}(\mathbf{p} = 0) = M_h$. For this reason, the correct procedure is to include both branches of the spectrum in the representation of the fluctuation field

$$h(x) = \Phi(x) - \langle \Phi \rangle \quad (66)$$

By expanding in eigenmodes of the momentum, it may be convenient, however, to separate out the component of the fluctuation associated with the long-wavelength modes Eq.(50), say $\tilde{h}(x)$, from the more conventional massive part of Eq.(49). The observable effects due to the excitation of \tilde{h} depend crucially on the value of c_s and will be discussed below.

3.4 Let us ignore, for the moment, all previous indications for a very large c_s and

just explore the phenomenological implications of long-wavelength modes in the spectrum as in Eq.(50). Whatever the value of c_s , these dominate the infrared region so that a general yukawa coupling of the Higgs field to fermions will give rise to a long-range *attractive* potential between any pair of fermion masses m_i and m_j

$$U_\infty(r) = -\frac{1}{4\pi c_s^2 \langle \Phi \rangle^2} \frac{m_i m_j}{r} \quad (67)$$

The above result would have a considerable impact for the Standard Model if we take the usual value $\langle \Phi \rangle$ related to the Fermi constant. Unless c_s be an extremely large number (in units of c) one is faced with strong long-range forces coupled to the inertial masses of the known elementary fermions that have never been observed. Just to have an idea, for $c_s = c$ the long-range interaction between two electrons in Eq.(67) is $\mathcal{O}(10^{33})$ larger than their purely gravitational attraction. On the other hand, invoking a phenomenologically viable strength, as if $c_s \langle \Phi \rangle$ were of the order of the Planck scale, is equivalent to re-obtain nearly instantaneous interactions transmitted by the scalar condensate as in Eq.(63).

Independently of phenomenology, a direct proportionality relation between $c_s \langle \Phi \rangle$ and M_{Planck} is also natural noticing that for $c_s \rightarrow \infty$ the energy-spectrum becomes Lorentz-covariant (with the exception of $\mathbf{p} = 0$). Therefore, in a picture where the ‘true’ dynamical origin of gravity is searched into long-wavelength deviations from exact Lorentz-covariance, it would be natural to relate the limit of a vanishing gravitational strength, $M_{\text{Planck}} \rightarrow \infty$, to the limit of an exact Lorentz-covariant spectrum, $c_s \rightarrow \infty$.

Returning to more phenomenological aspects, we observe that the potential in Eq.(67) can also be derived as a static limit from the effective lagrangian

$$\mathcal{L}_{\text{eff}}(\tilde{h}) = \frac{1}{2} \tilde{h} [\eta \Delta - \frac{\partial^2}{c^2 \partial t^2}] \tilde{h} - \frac{\tilde{h}}{\langle \Phi \rangle} \sum_f m_f c^2 \bar{\psi}_f \psi_f \quad (68)$$

where the free part takes into account the peculiar nature of the energy spectrum Eq.(50) and we have defined

$$c_s^2 \equiv \eta c^2 \quad (69)$$

Eq.(68) is useful to represent the effects of \tilde{h} over macroscopic scales as required by its long-range nature. The key-ingredient is the replacement of $m c^2 \bar{\psi} \psi$ with $T_\mu^\mu(x)$, the trace of the energy-momentum tensor of ordinary matter, a result embodied into the well known relation

$$\langle f | T_\mu^\mu | f \rangle = m_f c^2 \bar{\psi}_f \psi_f \quad (70)$$

If we neglect quantum fluctuations, this relation allows for an intuitive transition from the quantum to the classical theory. In fact, by introducing a wave-packet corresponding to a particle of momentum \mathbf{p} and normalization $\int d^3\mathbf{x}\bar{\psi}\psi = \frac{mc^2}{E(\mathbf{p})}$ we obtain

$$-mc^2 \int d^4x \bar{\psi}\psi = -mc \int ds \quad (71)$$

where $ds = cdt\sqrt{1 - \frac{\mathbf{u}^2}{c^2}}$ denotes the infinitesimal element of proper time for a classical particle with 3-velocity \mathbf{u} . Therefore, using the relation

$$\sum_n m_n c_n \int ds_n = \int d^4x T_\mu^\mu(x) \quad (72)$$

where

$$T_\mu^\mu(x) \equiv \sum_n \frac{E_n^2 - c^2 \mathbf{p}_n \cdot \mathbf{p}_n}{E_n} \delta^3(\mathbf{x} - \mathbf{x}_n(t)) \quad (73)$$

Eq.(68) is replaced by

$$\mathcal{L}_{\text{eff}}(\sigma) = \frac{1}{2} \tilde{h} [\eta \Delta - \frac{\partial^2}{c^2 \partial t^2}] \tilde{h} - \frac{\tilde{h}}{\langle \Phi \rangle} T_\mu^\mu \quad (74)$$

with the equation of motion

$$[\eta \Delta - \frac{\partial^2}{c^2 \partial t^2}] \frac{\tilde{h}}{\langle \Phi \rangle} = \frac{T_\mu^\mu}{\langle \Phi \rangle^2} \quad (75)$$

Finally, by comparing with Eqs.(27) and (29), we find

$$\frac{\tilde{h}(x)}{\langle \Phi \rangle} = -\sigma(x) \quad (76)$$

$$\frac{4\pi \tilde{G}_N}{c^2} = \frac{1}{\langle \Phi \rangle^2} \equiv G_F \quad (77)$$

and

$$S(x) = T_\mu^\mu(x) \quad (78)$$

In fact, for the large values of η that are suggested by the properties of the vacuum, the effects of \tilde{h} have practically no retardation effects. In this limit, Eq.(75) reduces to an instantaneous interaction

$$\Delta \frac{\tilde{h}}{\langle \Phi \rangle} = \frac{T_\mu^\mu}{\eta \langle \Phi \rangle^2} \quad (79)$$

of vanishingly small strength when $\eta \rightarrow \infty$. Finally for very slow motions, when the trace of the energy-momentum tensor becomes proportional to the mass density Eq.(16), one re-obtains, formally, the Poisson equation with the Newton constant G_N expressed as

$$G_N = \frac{G_F c^2}{4\pi \eta} \quad (80)$$

This fixes the values of η giving $c_s = \sqrt{\eta}c \sim 4 \cdot 10^{16}c$, as anticipated.

Assuming the above value for c_s , the relation (64) $\delta \sim \frac{M_h c^2}{c_s}$, and depending on the actual value chosen for $M_h c^2 = \mathcal{O}(\langle \Phi \rangle)$, one finds a length scale $R_{\text{mfp}} = \delta^{-1}$ in the millimeter range (~ 8 millimeters for $M_h c^2 = 1$ TeV). In this framework, the tight infrared-ultraviolet connection embodied in the relation $\delta \sim \frac{\langle \Phi \rangle^2}{M_{\text{Planck}}}$ is formally identical to that occurring in models [48] with extra space-time dimensions compactified at a size $R_c = R_{\text{mfp}}$.

For $r \sim R_{\text{mfp}} = \delta^{-1}$ the interparticle potential is not a simple $1/r$, as for asymptotic distances, but has to be computed from the Fourier transform of the h -field propagator

$$D(r) = \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p} \cdot \mathbf{r}}}{\tilde{E}^2(\mathbf{p})} \quad (81)$$

and depends on the detailed form of the spectrum that interpolates between Eqs.(50) and (49). In this picture, the millimeter range marks the typicale scale of ‘fifth-force’ experiments.

Notice that a similar picture would have been obtained by skipping subsects. (3.2)-(3.4) and simply identifying $\sigma(x)$ with an hypothetical ‘dilaton’ field \tilde{d} . This would be coupled to the trace of the energy-momentum tensor and replace our long-wavelength fluctuation field $\frac{\tilde{h}}{\langle \Phi \rangle}$. However, in this approach, the Newton constant has to be introduced from scratch as a fundamental scale that fixes the dilaton coupling. Alternatively, if the dilaton coupling is fixed by the Fermi scale G_F one has to solve the ‘hierarchy problem’, i.e. to replace our naturally, infinitely large c_s with some equivalent mechanism to explain the difference between G_F and G_N .

4 Summary and outlook

There are two different aspects of our analysis. On one hand, one can produce a simple picture of gravity in terms of a medium characterized by a scalar field $\sigma(x)$ that, for slowly varying fields, is known experimentally to coincide with the Newton potential.

On the other hand, independently of gravity, one is faced with the existence of a (non-Goldstone) gap-less mode of the Higgs field in the broken-symmetry phase. In fact, the simple perturbative idea of a purely massive singlet Higgs boson field depends on treating the scalar condensate as a classical c-number field. Beyond this level of approximation, $G^{-1}(p=0)$ is a two-valued function [34, 35] that includes the value $G^{-1}(p=0) = 0$, as in a gap-less theory. Exploiting the possible implications of this result should be very natural. After all, the Higgs field was introduced to obtain a consistent quantum theory and it would be really paradoxical to conclude that the Standard Model can only work provided we treat its vacuum state as

a purely classical c-number field. At the same time, if the Higgs vacuum is considered a real superfluid medium, made up of physical spinless quanta, it might even be *postulated* [49] that there are density fluctuations with an energy spectrum $\tilde{E}(\mathbf{p}) \sim c_s |\mathbf{p}|$ when $\mathbf{p} \rightarrow 0$. This would give rise to an attractive $1/r$ potential among all particles coupled to the (singlet) Higgs field that has to be understood. Alternatively, one can study the effect of Bose condensation on the forces among bodies sitting in a ‘ $\lambda\Phi^4$ ’ ambiance’ [50]. Below the transition temperature, i.e. in the broken-symmetry phase, the range of the forces becomes infinite and one finds a $1/r$ potential. Again, this shows that the condensate energy spectrum cannot be a pure $\sqrt{\mathbf{p}^2 + M_h^2}$ down to $\mathbf{p} = 0$ since, in this case, there would be no long-range force.

Just for this reason, looking for the physical origin of $\sigma(x)$, we propose a natural candidate: the collective density fluctuations of the scalar condensate. These propagate as longitudinal waves, starting at wavelengths that are larger than the mean free path R_{mfp} for the elementary phions and phenomenology requires R_{mfp} to be a length scale in the millimeter range. For this reason, there will be no variation of the gravitational potential between two points whose distance is smaller than R_{mfp} since the collective oscillations of the condensate have larger wavelengths. These average over distances that are much larger than the atomic size so that all quantum interference effects disappear, in agreement with the point of view expressed in the Introduction. An exception is represented by those particular experiments where the coherence of the wave-functions can be maintained over distances where $\sigma(x)$ can vary appreciably, as for the neutron diffraction experiments in the earth’s gravitational field [51]. In such cases, an acceleration transformation to a suitable freely falling frame, to eliminate the effects of the direct coupling of $\sigma(x)$ to all particles, will introduce mass-dependent phases for the various wave-functions. Therefore, in principle, through quantum interference experiments, one might distinguish between the two frames.

The vastly superluminal value of the ‘sound velocity’ $c_s = \sqrt{\eta}c$, with $\sqrt{\eta} = 4 \cdot 10^{16}$, needed to relate the Newton constant G_N to the Fermi constant G_F , is the only free parameter of our analysis. However, its magnitude is not totally unexpected on the base of the properties of the Higgs condensate. Indeed, it is consistent with *three* different arguments, all pointing toward $c_s \rightarrow \infty$, that would motivate the mysterious, nearly instantaneous nature of Newtonian gravity [52] at the base of its traditional interpretation as an ‘action at distance’. This picture provides a simple physical solution of the ‘hierarchy problem’ where the large value of the dimensionless ratio ($\delta \sim R_{\text{mfp}}^{-1}$)

$$\sqrt{\eta} \sim \frac{M_h}{\delta} \sim \frac{M_{\text{Planck}}}{M_h} = \mathcal{O}(10^{16}) \quad (82)$$

derives from the Lorentz-covariance of the energy spectrum in the cutoff theory down to $|\mathbf{p}| \sim \delta$ (see Eq.(64)), with $\frac{\delta}{M_h} \rightarrow 0$ in the continuum limit.

It is interesting that the various quantities entering Eq.(82) admit a simple interpretation in terms of the two basic quantities of the phion condensate: the phion density n and the phion scattering length a . In terms of these quantities we find [5] $M_h^2 \sim na$ and $R_{\text{mfp}} \sim \frac{1}{na^2}$ [40]. Therefore, using our results from subsect.3.4, we find

$$M_h R_{\text{mfp}} \sim \frac{1}{\sqrt{na^3}} = \mathcal{O}(10^{16}) \quad (83)$$

with a scattering length of the order of the Planck length

$$a \sim na^3 R_{\text{mfp}} \sim \mathcal{O}(10^{-33}) \text{ cm} \quad (84)$$

since the ‘diluteness factor’ $na^3 \sim \mathcal{O}(10^{-33})$ is extremely small.

Before concluding, we shall mention some points that deserve further study:

i) the idea of the Higgs condensate as a real superfluid medium suggests alternative scenarios for the Higgs boson production, in addition to the standard mechanisms (e.g through quark pair, W-pair,...annihilation). For instance, considering high-energy cosmic ray proton-proton collisions, the whole energy content of the initial state $\sqrt{s} \sim \sqrt{2m_p E}$ might be used for a ‘local heating’ of the vacuum [53]. A substantial heat release might drastically excite the superfluid vacuum and give rise to dissipative processes, analogously to the shock waves produced by a moving body with current density $|\mathbf{J}(\mathbf{r})| > c_s n$. If we identify $M_h c^2 \sim c_s \delta$ as the relevant energy scale, we would tentatively conclude that the local-heating mechanism might become efficient for center of mass energies $\sqrt{s} > M_h c^2$ or for cosmic ray energies $E > 5x^2 \cdot 10^{14} \text{ eV}$ where x is the value of $M_h c^2$ in TeV. In this case, a sizeable fraction of the primary flux might be used to excite the vacuum, rather than to produce leading hadrons with the associated electromagnetic showers. Therefore, on the ground, one would count less events, as if the primary flux would have been reduced, and one might speculate on alternative interpretations of the famous ‘knee’ [54] in the cosmic ray spectrum whose precise position and physical origin are still unclear [55]. For instance, early investigations [56] showed a rather sharp peak at $E \sim 5 \cdot 10^{15} \text{ eV}$ whereas newer measurements [57, 58] favour a more gradual steepening starting at $E \sim 1 - 2 \cdot 10^{15} \text{ eV}$. At the same time, there may be some inconsistencies in the standard astrophysical interpretations [55, 61] that could motivate the idea [59] that some new state (“...strongly interacting bosons with masses $> 400 \text{ GeV}/c^2$..” [60]) is indeed produced in the primary collision.

ii) although all metrics discussed by Tupper agree in the weak-field limit [27], we have explained why Yilmaz's metric would play a special role: in this case, the metric depends on the scalar field $\sigma(x)$ in a parametric form so that Einstein's field equations are actually algebraic identities. This is a consistency requirement in a theory where gravity is an effective interaction induced by the vacuum of some underlying quantum field theory. However, Yilmaz's theory differs from General Relativity for strong fields where the Schwarzschild singularity $\frac{1}{1-2\sigma}$ becomes an exponential form $e^{2\sigma}$. For a many-body gravitational system, the unique factorization properties of the Yilmaz metric, $e^{\sum_i \frac{M_i}{r_i}} = e^{\frac{M_1}{r_1}} e^{\frac{M_2}{r_2}} \dots$, provide an alternative explanation for the controversial huge quasar red-shifts, a large part of which could be interpreted as being of gravitational (rather than cosmological) origin [62]. This might represent a 'fifth' test of gravity outside of the weak-field regime.

iii) in our approach, T_μ^μ is the source of the long-wavelength density fluctuations of the Higgs condensate and, as such, of Newtonian gravity. This is a specific consequence of the coupling to a scalar field. In this sense, the Higgs vacuum provides the natural framework for a common dynamical origin of inertia and gravity that would both disappear without the scalar condensate, i.e. in the limit $\langle \Phi \rangle \rightarrow 0$. It also provides an explicit realization [49] of the *Mach's Principle* where the vastly superluminal value of c_s might represent the non-local element to understand the apparent a-causal nature of the inertial reactions in an accelerated frame [63].

iv) as discussed by Dicke [32], when averaged over sufficiently long times (e.g. with respect to the atomic times), by the virial theorem [64], the integral of T_μ^μ represents the total energy of a bound system, i.e. includes the binding energy. Therefore, for microscopic systems whose components have large v^2/c^2 but very short periods, this definition becomes equivalent to the rest energy. On the other hand, for macroscopic systems, that have long periods but small v^2/c^2 , there should be no observable differences from the mass density. A possible exception might be associated with the overall motion of our galaxy, when assuming for this velocity the sizeable value $\frac{v}{c} \sim 10^{-3}$ [32]. This velocity affects all known classical sources of gravity and should represent an overall re-definition of their mass. However, small differences may eventually be found by precise observations of the GPS satellites. These are placed on nearly circular orbits of radius $r_{\text{GPS}} \sim 26600$ Km and periods $T_{\text{GPS}} \sim 11$ hours and 58 minutes [10]. If the trace of the energy-momentum tensor is the true source of gravity, the GPS Keplerian 'invariant' $T \cdot r^{-3/2}$ should change by about one part over 10^7 when the earth's velocity is parallel or antiparallel to the galactic velocity. Although small, this effect

could be observable in view of the spectacular accuracy of the GPS system.

v) one should not conclude that the (nearly) instantaneous nature of Newtonian gravity, in our picture, is in contradiction with binary pulsar data showing that gravity is due to "... massless spin-2 gravitons propagating at the speed of light". In fact, the observed slowing down of the binary systems, when interpreted in terms of a quadrupole gravitational radiation, could at best be used to predict *deviations* from exact Keplerian orbits and not the Keplerian orbits themselves. This means that, in principle, the slowing down of binary pulsars and the physical mechanism for Newtonian gravity represent separate issues whose possible conceptual unification depends on the theoretical framework. In General Relativity, unification consists in introducing the same entity, the graviton field $h_{\mu\nu}$, with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{44} = \frac{2M}{r}$ is used for the Newton potential and the transverse components account for gravitational radiation. However, condensed media are full of examples where longitudinal and transverse excitations do not propagate with the same velocity. In addition, it is now well known [65] that the standard D'Alembert wave equation can exhibit superluminal solutions (there are even some *experimental* evidences [66]). This can also be checked following our own simple argument. Let us define $\eta = z - ut$ and $r = \sqrt{\frac{u^2 - c^2}{c^2}} \sqrt{x^2 + y^2}$ where $u^2 > c^2$. Any function $\psi = \psi(\eta, r)$ describes 'cylindrical waves' propagating along the z-axis with a velocity $u > c$ since ψ has the same value for $z - ut = \text{const}$ and $x^2 + y^2 = \text{const}$. However, one can find special types of ψ 's that are solutions of the D'Alembert wave equation, i.e. for which

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{c^2 \partial t^2}\right)\psi = 0 \quad (85)$$

In fact, with our choice of ψ , Eq.(85) becomes the Darboux equation

$$\left(\frac{\partial^2}{\partial \eta^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r}\right)\psi = 0 \quad (86)$$

whose general solution [67] can be given as

$$\psi(\eta, r) = \int_{-1}^1 \frac{f(\eta + r\mu)d\mu}{\sqrt{1 - \mu^2}} \quad (87)$$

in terms of an arbitrary $f = f(q)$ that is a twice-differentiable function of its argument. Since the superluminal $\psi(\eta, r)$ are not standard plane-wave solutions $\psi^{(o)} = \psi^{(o)}(z - ct)$, longitudinal gravitons might propagate differently from transverse gravitons, although being both solutions of a D'Alembert wave equation.

vi) on the other hand, by considering General Relativity with a cosmological constant, it is even less clear why gravitons should be ordinary massless particles (i.e. similar to

physical spin-1 photons). In fact, a non-zero cosmological constant λ acts as a graviton mass term in a linearized approximation where one replaces $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, $\bar{g}_{\mu\nu}$ being a suitable background solution of Einstein equations with a cosmological term λ . This observation, that to our knowledge has only been raised in ref.[68], amounts to the following. Let us first consider the original static Einstein universe. Using the same notations of [69] (with $G_N = c^2 = 1$) the field equations are

$$R_{ik} - \frac{1}{2}g_{ik}R - \lambda g_{ik} = -8\pi T_{ik} \quad (88)$$

For $T_{ik} = \mu_o \delta_4^i \delta_4^k$, where μ_o is the constant value of the mass density and $\lambda = \frac{1}{a^2} = 4\pi\mu_o$, the solutions are

$$\bar{g}_{ik} = \delta_k^i + \frac{x^i x^k}{a^2 - [(x_1)^2 + (x_2)^2 + (x_3)^2]} \quad (i, k = 1, 2, 3) \quad (89)$$

and $\bar{g}_{i4} = 0$, $\bar{g}_{44} = -1$. Now let us consider, in this universe, a point-like mass perturbation by replacing $\mu_o \rightarrow \mu_o + M\delta^{(3)}(\mathbf{r})$ and look for the leading order correction $g_{44} = \bar{g}_{44} + h_{44}$ that leads, around flat space, to $h_{44} = \frac{2M}{r}$. Here, however, the equation for h_{44} is

$$(\Delta + \frac{6}{a^2})h_{44} = -8\pi M\delta^{(3)}(\mathbf{r}) \quad (90)$$

Its solution is *not* equivalent to replace the Newton potential with a Yukawa potential $\sim \frac{e^{-\sqrt{\lambda}r}}{r}$ (as Pauli says). Instead, we find an oscillatory potential $\frac{e^{\pm ipr}}{r}$, where $p^2 = \frac{6}{a^2}$, and an imaginary graviton mass $m_g = ip$ whose physical interpretation is unclear, as for a ‘wrong-sign’ Klein-Gordon equation. Similar features are found in the Gödel universe [70, 71] where the presence of the cosmological term gives rise to closed time-like loops, or in the presently expanding-universe scenarios where a cosmological term is also needed to match the experimental observations. In the latter case, the linearized problem should be cast in the form

$$(\hat{M}_{\mu\nu}^{\alpha\beta}(\bar{g}) + \delta_\mu^\alpha \delta_\nu^\beta \lambda)h_{\alpha\beta} = 0 \quad (91)$$

where the graviton ‘mass matrix’ depends on the actual background metric and background energy-momentum tensor used to fit the cosmological data. For this reason, independently of any experiment to detect gravitational waves, the simple idea of massless gravitons propagating, just as real photons, at the speed of light is far from being obvious. Finally, it was pointed out by Segal [72] that the Einstein universe provides a very convenient covering space of Minkowski space-time, preserving the global causality structure of conformal-invariant wave equations (Maxwell equations, massless $\lambda\Phi^4$, Yang-Mills,...). Therefore, at least in the case

of the Einstein universe, the apparent unphysical nature of its gravitons may just indicate the failure of the linearized approximation.

vii) as for the relation between the longitudinal density fluctuations and the Newtonian potential, identifying in the Higgs condensate genuine transverse degrees of freedom could help to place the analogy with General Relativity on a much tighter base. To this end, comparing with superfluid ^4He , it would be natural to consider quantized vortices that can support circularly polarized transverse vibrations [73]. In particular, Feynman's treatment [74] explains very well how vortex rings of suitable size are formed and can propagate almost freely, in the superfluid. This is due to the invariance of the superfluid wave function for a permutation of the atoms and does not correspond to a physical, real circulation. In this sense, the remaining part of the fluid plays no role, as it would be for an ordinary vacuum state. This is suprisingly close to Lorentz's picture [75] of extended elementary particles that are "...some local modifications in the state of the aether. These modifications may of course very well travel onward while the volume-elements of the medium in which they exist remain at rest." For this reason, quantized vortex rings in the Higgs condensate might represent the 'superfluid equivalent' of circularly polarized transverse gravitons, just as the vortices invented by Thomson [76] to represent circularly polarized transverse photons. About the possible radius of the rings R and their transverse dimension d , using Eq.(84), there is a very wide range of d and R , say

$$a \leq d \ll R < R_{\text{mfp}} \quad (92)$$

for which phions undergo no observable scattering process inside the ring. Therefore, following Bloch's treatment [77] of superfluid rings, where the elementary phions replace the ^4He atoms, and considering the ring quantized angular momentum, $L = \nu\hbar$, one could try to associate circularly polarized graviton states to the values $\nu = \pm 2$.

viii) additional connections with General Relativity arise when relating vortex rings to strings. To this end, there are two possibilities. On one hand, a tower of vortex rings, piled along the z -axis and with an infinitesimal radius in the (x, y) plane, provides a physical representation of an open string with a well defined angular momentum J per unit length along the z -axis. When solving Einstein equations for such a field configuration [71], even for zero cosmological constant, the resulting geometry supports time-like loops (suitable circular paths around the string in the (x, y) plane) thus re-proposing the problem of causality in the presence of a scalar condensate. On the other hand, a vortex ring of infinitesimal transverse section can also be considered a closed string whose possible excitation states, as derived from

Bloch's picture [77], require an infinite set of quantum numbers. This is also independent of the nature of the elementary quanta. For instance, replacing the ^4He atoms with Cooper pairs leads to a similar picture [77], in agreement with the analogy between scalar and fermion superfluid vacua mentioned in the Introduction. By considering superfluid rings as elementary objects, one could try to construct an effective lagrangian and possible forms of ring-ring interactions. In this context, it is interesting that the spontaneously broken phase of a one-component $\lambda\Phi^4$ theory, in four space-time dimensions, has a non-trivial duality mapping [78] into a theory of interacting membranes whose continuum limit is the Kalb-Ramond model [79]. This duality transformation could help to connect the operators that excite the (st)ring-like degrees of freedom to the basic annihilation and creation operators for the elementary scalar quanta, in analogy with the more conventional Bogolubov transformation from particle to phonon quasi-particle states. We end up, by mentioning that Bloch considers the possibility to account for the required interaction between ^4He atoms through their mere replacement in the ring by the phonon excitations of the superfluid, at least for low values of the associated 3-momentum where these behave as non-interacting quasi-particles. This means that, in addition to the microscopic rings of Eq.(92), there might be a whole second generation of 'cosmic' (st)rings, whose radial size could extend up to the length scale associated with a free-phonon propagation.

ix) this last remark suggests to look for deviations from the free-phonon propagation that represents, in our picture, the mechanism for Newtonian gravity. To this end, one should estimate the value of a mean free path associated with phonon propagation. In superfluid ^4He , for temperature $T \rightarrow 0$, the phonon mean free path becomes larger than the size of the container. However, in a real infinite system, one should consider its possible effects. Notice that the phonon mean free-path is much larger than the phion mean free path $R_{\text{mfp}} \sim \frac{1}{na^2}$ that we have considered so far. In fact, the former depends on the *residual* interactions in our infinitely diluted hard-sphere system where, at zero-temperature, the phonon spectrum is almost exact. In fact, the residual interactions are due to a non-zero depletion n_D , the residual fraction of particles that, at zero-temperature and in the absence of any external field, are not in the same condensed quantum state $\mathbf{p} = 0$. In the phion condensate where $\sqrt{na^3} = \mathcal{O}(10^{-16})$, this fraction $\frac{n_D}{n} \sim \sqrt{na^3}$ [41] is infinitesimal and can be taken as a measure of the residual interactions. In this sense, the relevant density to determine the phonon mean free path is n_D and not n . Therefore, we would tentatively estimate [49] a phonon mean free

path of

$$\zeta_{\text{mfp}} \sim \frac{1}{n_D a^2} \sim 4 \cdot 10^{16} R_{\text{mfp}} \quad (93)$$

to mark the distance over which non-linear effects might modify the free-phonon propagation and, thus, Newtonian gravity. For our standard value $R_{\text{mfp}} = 8$ millimeters, we find $\zeta_{\text{mfp}} \sim 3 \cdot 10^{16} \text{ cm}$ or $2 \cdot 10^3 \text{ AU}$. Checking this prediction would require to detect some anomaly in the behaviour of long-period comets, those with $T > 200 \text{ yr}$ and semimajor axis larger than $\sim 10^2 \text{ AU}$. These are believed to originate from the Oort cloud [80], a roughly spherical condensate of $\sim 10^{13}$ comets of average mass $\sim 10^{16}$ grams [81] placed between $10^{3.5}$ and $10^{4.5} \text{ AU}$ [82]. To describe some of their features one has to introduce some ‘ad hoc’ assumptions [82], and this might be indicative of deviations from a pure Newtonian behaviour. As a possible model for the deviations, we would tentatively follow the modification of Newtonian dynamics (‘MOND’) proposed by Milgrom [83] as an alternative approach to the mass discrepancy in galactic systems. In fact, this is not a pure long-distance modification of gravity. Rather, it is a modification of inertia and/or gravity that shows up in the unusual deep-space regime of very low acceleration, of the order of $g_o \sim 10^{-8} \text{ cm} \cdot \text{sec}^{-2}$. In this sense, MOND is the natural type of effect one might expect in our picture where gravity originates as a collective oscillation of the same medium that generates inertia. For instance, in Milgrom’s approach, a long-term comet seen around the sun is bound [84] when its total energy $E = \frac{v^2}{2} - \frac{G_N M_{\text{sun}}}{r}$ is below a condensation shell $E < \sqrt{G_N M_{\text{sun}} g_o}$ and not only when $E < 0$. If we attempt the identification

$$g_o \sim \frac{G_N M_{\text{sun}}}{\zeta_{\text{mfp}}^2} \quad (94)$$

we obtain

$$\zeta_{\text{mfp}} \sim 10^{17} \text{ cm} \quad (95)$$

which is not too far from our reference value $3 \cdot 10^{16} \text{ cm}$. For a closer contact with Milgrom’s approach, we note that, on the base of the general arguments presented at the beginning of subsect.3.3, the deviations from free-phonon propagation correspond to observable changes in the phion density n . These can formally be included following the treatment of quantum liquids [85]. In this case, the free-phonon approximation is equivalent to a free Lagrangian density

$$\mathcal{L}_o = -\frac{1}{2}(\nabla\sigma)^2 - \sigma\rho \quad (96)$$

that upon minimization provides the Poisson equation. Following [85], the residual interac-

tions amount to replace Eq.(96) with

$$\mathcal{L} = -\frac{1}{2}\frac{n}{n_o}(\nabla\sigma)^2 - \mathcal{W}(n) - \sigma\rho \quad (97)$$

i.e. allowing for $n \neq n_o$ and introducing an internal energy $\mathcal{W}(n)$. In this case, upon minimization one finds

$$\nabla\left(\frac{n}{n_o}\nabla\sigma\right) = \rho \quad (98)$$

and

$$\frac{1}{2n_o}(\nabla\sigma)^2 + \frac{d\mathcal{W}}{dn} = 0 \quad (99)$$

Finally, after deriving $n = n(|\nabla\sigma|)$ from Eq.(99), one has to introduce a constant acceleration g_o in order the function $\mu = \frac{n}{n_o}$ to be dimensionless. Therefore, using typical techniques of quantum field theories [86], one may attempt a microscopic derivation of Milgrom's function $\mu = \mu(\frac{|\nabla\phi|}{g_o})$ used in his non-linear description of gravity

$$\nabla\left(\mu\left(\frac{|\nabla\phi|}{g_o}\right)\nabla\phi\right) = \rho \quad (100)$$

In fact, as discussed in ref.[87], this has the same form of the stationary flow of an irrotational fluid of suitable density. However, it can also be interpreted as the gravitational analog of a dielectric medium where $\nabla(\epsilon(|\mathbf{E}|)\mathbf{E}) = \rho_e$ with $\mathbf{E} = -\nabla U$, U being the electrical potential. This type of correspondence is very natural in our approach where gravity originates from the density fluctuations of the scalar condensate.

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